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COMBINED IMPACT OF INTERNAL HEATING AND VARIABLE VISCOSITY ON THE ONSET OF BENARD-MARANGONI DOUBLE DIFFUSIVE CONVECTION IN A BINARY FLUID LAYER

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ABSTRACT

The present article is to examine the joint influence of internal heating and variable viscosity on the onset of double diffusive convective motion using perturbation technique. Linear stability analysis is performed and it is assumed that the upper surface of a fluid layer is deformably free and that viscosity dependency is believed to be exponential. The boundaries are known to be rigid, but permeable, and insulated to fluctuations in temperature. From results of an increasing the viscosity parameter, the system shows destabilizing effect and the system will be stabilized by that internal heat source power. It is also revealed the influence of double diffusive coefficients. The effect of the thermal diffusion is found to have a destabilizing reaction on the system, whereas the opposite reaction is noted with an increase of thermo-diffusion parameter.

KEYWORDS: Variable Viscosity, Internal Heat Source & Benard-Marangoni Convection

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1. INTRODUCTION

Dufour diffusion, also known as thermo-diffusion, and Soret diffusion, also known as thermal diffusion, is significant in both non-Newtonian and Newtonian heat convection and mass shifts and are often found in high-speed aerodynamics and chemical process engineering. Such findings are important in combining heat and mass transfer for the intermediate molecular weight gasses in binary's. Hurle and Jakeman (1971) demonstrated the thermosolutal convection induced by Soret using a methanol-water mixture, both experimentally and theoretically. Instead, using a salt solution, Caldwell (1970) extended the study, while Platten and Chavepeyer (1973) continued previous studies using a combination of water-ethanol. The linear stability of experimental thermal diffusion convection in an ethanol-water mixture was investigated by Knobloch and Moore (1988) under various boundary conditions with a focus on the Biot number. Taking into account the Soret interaction with other physical causes, the thermo capillary instability in a binary fluid at the start of convection was studied. (Bergeon et al. (1988), Slavtchev et al.(1991). Saravanan and Sivakumar (2009)). Chand and Rana (2015) recently studied the Soret effect in the presence of a vertical magnetic field in a nanofluid layer.

Some researchers assume that fluid has a constant viscosity or may possess temperature-dependent viscosity (viscosity decreases exponentially with temperature) which may affect the stability convection. Palm (1960) initiated the study of variable viscosity on a steady convection. Other researchers also studied the variable viscosity effects in different problems where Torrance and Turcotte (1971) and Stengel et al. (1982) studied in Benard instabilities and Slavtchev et al. (1999), Cloot and Lebon (1985) and Kozhoukharova and Roze (1999) in

Marangoni instabilities. Cloot and Lebon (1985) studied the steady Marangoni convection with undeformable surface and in microgravity. Abidin et al. (2008) and Arifin and Abidin (2009a,2009b) studied the temperature-dependent viscosity effect together with others effects such as the feedback control, deformable surface and boundary effect in a fluid layer. White (1998) did an overall study where he studied both theoretical and experimental in a Benard convection meanwhile Manga et al. (2001) compared the temperature effect experimentally with boundary layer models.

Temperature dependent viscosity also has been integrated into other convection system where Franchi and Straughan (1992) included the effect in a micropolar uid and Ramirez and Saez (1990) in a porous medium. It stated that a higher temperature in both systems will cause the critical Rayleigh number to decrease. Ramirez and Saez (1990) stated that temperature-dependent viscosity should be taken into account for every case studied since the effect has a huge impact on the instability of convection. A similar result obtained by Lu and Chen (1995) where the stability was enhanced by decreasing the temperature.

Friedrich and Rudraiah (1984) have shown that the convection can be suppressed with a sufficient non-uniform temperature gradient. Bhattacharyya and Jena (1984) found a destabilizing effect of the heat source. Char and Chiang (1994) showed that internal heat generation has a significant impact on the stability of Benard–Marangoni convection. Through a rigorous investigation, they found that the system is destabilized by an increase in internal heat generation. Capone et al.(2011) investigated the impact vertical through flow on the onset of double-diffusive penetrative convection. In a rotating anisotropic porous sheet, Bhadauria et al. (2011) studied the influence of internal heat generation on the onset of natural convection using a weak nonlinear model. Very recently, Yadav *et al.* (2012,2015) and Wakif *et al.*(2016) examined the effect of internal heat source in a nanofluid layer.

In a fluid layer with internal heating, double-diffusive convection despite its value in many hands-on industries. The current findings attempt the effects of internally heating fluid layer in the presence of thermo and thermal diffusion. We perform a stability analysis, and the obtained eigen value problem is solved using the perturbation procedure.

2. CONCEPTUAL MODEL

Two horizontal layers of quiescent double diffusive binary fluid with thickness d is heated from below where the temperature difference is represented by ΔT. The upper free surface of the fluid layer with its location being free of deformities $z = d + \Omega(x, y, t)$ and the z-axis pointing vertically upwards opposite the direction of gravity. For a Boussinesq approximation, we assumed the constant physical fluid properties except the density, ρ and surface tension, σ (for Marangoni convection) to vary upon temperature, T and solute concentration, S Chen and Su(1992) and takes the form

$$\sigma = \sigma_0 - \sigma_t (T - T_0) + \sigma_s (S - S_0) \tag{1}$$

$$\rho = \rho_0 [1 - \rho_t (T - T_0) + \rho_s (S - S_0)] \tag{2}$$

Here, σ_0 and ρ_0 are the values at the reference concentration, S_0 and the reference temperature, T_0. σ_t and ρ_t are the rate of change with temperature and σ_s and ρ_s are the rate of change of density with concentration. Due to the

temperature-dependent viscosity in a binary mixture, the kinematic viscosity, μ follows Hilt *et al.* (2014) in the form

$$\mu = \mu_0 \exp\left[-A\left(T - T_0\right)\right] \tag{3}$$

and μ_0 is the dynamic viscosity corresponding to a temperature equal to the mean of temperature at the boundaries.

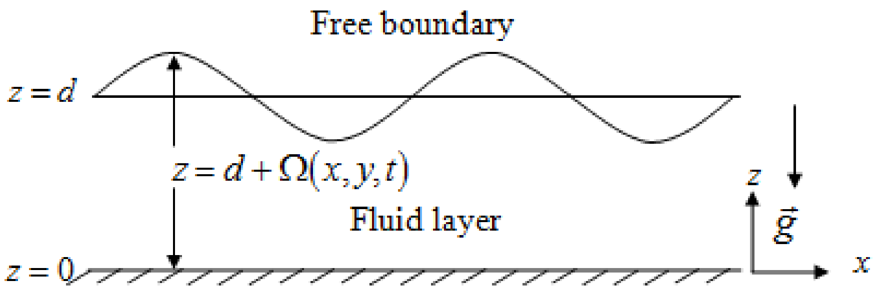

Figure 1: Physical Configuration.

3. MATHEMATICAL FORMULATION

The governing equations of motion are:

$$\nabla \cdot \vec{V} = 0 \tag{4}$$

$$\rho_0 \left(\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \nabla\right)\vec{V} \right) = -\nabla p + \rho_0 \vec{g} + 2\nabla . \left[\mu \left(\nabla . \vec{V} + \nabla . \vec{V}^T \right) \right] \tag{5}$$

$$\frac{\partial T}{\partial t} + \left(\vec{V} \cdot \nabla\right) T = \kappa \nabla^2 T + D_{TS} \nabla^2 T + q \tag{6}$$

$$\frac{\partial S}{\partial t} + \left(\vec{V} \cdot \nabla\right) S = \kappa_S \nabla^2 S + D_{ST} \nabla^2 T + q \tag{7}$$

where the variables are represented as follows; $\vec{V} = (u, v, w)$ is the velocity, S (solute concentration), p (pressure), k (thermal diffusivity), k_s (solutal diffusivity), D_{TS} (Soret diffusivity) and lastly, D_{ST} is the Dufour diffusivity. The basic state of the fluid is

$$(u, v, w, \rho, T, p, \mu, S) = \left[0, 0, 0, \rho_b(z), T_b(z), p_b(z), \mu_b(z), S_b(z)\right] \tag{8}$$

Infinitesimal disturbances are implemented to test the stability of the basic solution

$$(u, v, w, \rho, T, p, \mu, S) = \left[0, 0, 0, \rho_b(z), T_b(z), p_b(z), \mu_b(z), S_b(z)\right] + \left[u', v', w', \rho', T', p', \mu', S'\right] \tag{9}$$

And using equations (8) and (9), we obtain

$$\frac{1}{pr}\frac{\partial}{\partial t}\nabla^2 w=\tilde{f}\,\nabla^4 w+2\frac{\partial\tilde{f}}{\partial z}\nabla^2\frac{\partial w}{\partial z}+\frac{\partial^2\tilde{f}}{\partial z^2}\left(\nabla^2 w-2\nabla_h^2 w\right)+R\nabla_h^2 T+Rs\nabla_h^2 S \tag{10}$$

$$\frac{\partial T}{\partial t}=\nabla_h^2 T+D_F\,\nabla_h^2 S+w\left[1-Ns(1-2z)\right] \tag{11}$$

$$\frac{\partial S}{\partial t}=Sr\,\nabla_h^2 T+Le\,\nabla_h^2 S+w \tag{12}$$

Where $R=\dfrac{\alpha g\,\Delta T d^3}{\nu\,\kappa}$ (Rayleigh number), $Rs=\dfrac{\rho_s g\,\Delta S d^3}{\nu\,\kappa_s}$ (Solutal Rayleigh) number, $Ns=q\,d^2/2\kappa\left(T_0-T_u\right)$ (dimensionless heat source strength), $Le=\dfrac{\kappa_s}{\kappa}$ (Lewis number), $Sr=\dfrac{D_{ST}\,\Delta S}{\kappa\Delta T}$ (Soret number) and $D_F=\dfrac{D_{TS}\,\Delta S}{\kappa\Delta T}$(Dufour number) *and* $Pr=\dfrac{\nu}{\kappa}$ (Prandtl number). $\nabla^2=\nabla_h^2+\partial^2/\partial z^2$ is the Laplacian operator with $\nabla_h^2=\partial^2/\partial x^2+\partial^2/\partial y^2$. The function $\tilde{f}$ representing variable viscosity function, is defined as

$$\tilde{f}=\exp\left[B\left(z-\frac{1}{2}\right)\right],\quad B=\left(\frac{\nu_{\max}}{\nu_{\min}}\right). \tag{13}$$

The respective boundary conditions are

$$\frac{\partial w}{\partial z}=0,\ w=0\ \text{and}\ \frac{\partial T}{\partial z}=0\qquad \text{at}\ z=0 \tag{14}$$

$$\frac{\partial\Omega}{\partial t}=w,\frac{\partial T}{\partial z}+Bi\left[T-\left(Ns+1\right)\Omega\right]=0\qquad at\quad z=1 \tag{15}$$

$$\tilde{f}\left(\frac{\partial^2}{\partial z^2}-\nabla_h^2\right)w=M\,\nabla_h^2\left[T-\left(1+Ns\right)\Omega\right]\qquad at\quad z=1 \tag{16}$$

$$\tilde{f}\,Cr\left[\frac{1}{\Pr}\frac{\partial}{\partial t}+\left(\frac{\partial^2}{\partial z^2}+3\nabla_h^2\right)\right]\frac{\partial w}{\partial z}+\left(B_0-\nabla_h^2\right)\nabla_h^2\Omega=0\qquad at\quad z=1 \tag{17}$$

Because the theory of exchange volatility also holds true for the present configuration, the time derivatives would also be removed from Eqs (10) and (11) respectively. Then the dependent variables will be expanded in normal mode as

$$\left(w,T,S\right)=\left[W\left(z\right),\theta\left(z\right),\Phi\left(z\right)\right]\ \exp\left[i\left(lx+my\right)\right] \tag{18}$$

Then, we get the following differential equations for eigen value problem

$$\tilde{f}\left(D^2-a^2\right)^2 W+2D\tilde{f}\left(D^2-a^2\right)DW+D^2\tilde{f}\left(D^2+a^2\right)=Ra^2\theta+\frac{1}{Le}Rs\,a^2\Phi \tag{19}$$

$$\left(D^2-a^2\right)\theta+D_F\left(D^2-a^2\right)\Phi=-W\left[1-Ns\left(1-2z\right)\right] \tag{20}$$

$$Sr\left(D^2-a^2\right)\theta+Le\left(D^2-a^2\right)\Phi=-W \tag{21}$$

The linearized boundary conditions are:

$$W=D\theta+Bi\left[\theta-\left(1+Ns\right)Z\right]=0 \qquad at \quad z=1 \tag{22}$$

$$\tilde{f}\left(D^2+a^2\right)W+Ma^2\left[\theta-\left(1+Ns\right)Z\right]=0 \qquad \text{at} \quad z=1 \tag{23}$$

$$\tilde{f}\,Cr\left(D^2-3a^2\right)DW=\left(B_0+a^2\right)a^2Z \qquad \text{at} \quad z=1 \tag{24}$$

$$W=0, \quad DW=0 \text{ and } D\theta=0 \qquad \text{at } z=0 \tag{25}$$

4. ANALYTICAL SOLUTION

Equations (19)-(21) can be solved analytically using regular perturbation technique $\left(RPT\right)$, subjected to boundary conditions, equations (22)-(25). The variables are expressed in terms of the small wave number to study the validity.

$$\left(W,\theta,\Phi\right)=\sum_{i=0}^{N}\left(a^2\right)^i\left(W_i,\theta_i,\Phi_i\right) \tag{26}$$

Substitution of Eq.(26) into Eqs.(19)-(21) and the boundary conditions (22) - (25)

$$\tilde{f}\,D^4W_0+2D\tilde{f}\,D^3W_0+D^2\tilde{f}\,D^2W_0=0 \tag{26}$$

$$D^2\theta_0+D_FD^2\Phi_0=-f(z)W_0 \tag{27}$$

$$Sr\,D^2\theta_0+Le\,D^2\Phi_0=-W_0 \tag{28}$$

where

$$f(z)=\left[1-Ns\left(1-2z\right)\right] \tag{29}$$

The required boundary conditions (22) - (25), takes the form

$$W_0=DW_0=D\Theta_0=0 \qquad at \quad z=0 \tag{30}$$

$$\tilde{f}(1)D^2W_0=D\Theta_0=0 \qquad at \quad z=1 \tag{31}$$

Then solutions for equations above are

$$\Theta_0=1 \quad \text{and} \quad W_0=0 \tag{32}$$

First- order equations (19) - (21) become

$$D^4W_1 + 2B\ D^3W_1 + B^2\ D^2W_1 = \left(R + \frac{1}{Le}Rs\right) Exp[-B\left(z - 1/2\right)] \tag{33}$$

$$D^2\theta_1 = 1 + D_F - f(z)W_1 \tag{34}$$

$$D^2\Phi_1 = Sr + Le - W_1 \tag{35}$$

The boundary conditions (22) - (25) become

$$W_1 = DW_1 = 0 \quad at \quad z = 0 \tag{36}$$

$$\tilde{f}(1)D^2W_0 + Ma^2\left[\Theta - (1 + Ns)Z\right] = 0 \quad at \quad z = 1 \tag{37}$$

$$\tilde{f}(1)D^3W_1 - \frac{B_0}{Cr}Z_0 = 0 \ at \ \ z = 1 \tag{38}$$

The general solution of (33) is

$$W_1 = \left(R + \frac{1}{Le}Rs\right)\left[C_1 + C_2 z + C_3 e^{-Bz} + C_4\ ze^{-Bz} + \frac{z^2}{2B^2} Exp[-B\left(z - 1/2\right)]\right] \tag{39}$$

where

$$C_1 = -\frac{e^{-B/2}}{2B^2}\left(\frac{22e^{-B} + B - 7}{10 - 5e^{-B} + 5e^{-2B} - 7B^3e^{-B}}\right),\ C_2 = \frac{e^{-B/2}}{-2B^2}\left(\frac{2 - 21e^{B} + 3B + 11Be^{B}}{7e^{2B} - B^3e^{B} - B}\right)$$

$$C_3 = \frac{e^{B/2}}{2B^3}\left(\frac{e^{-B} - B^2 - 1}{1 + 4e^{2B} - 4e^{B} - B^3e^{B}}\right),\ C_4 = \frac{e^{B/2}}{2B^2}\left(\frac{2 - 21e^{B} + 12B^3 + B^2}{22e^{B} - 3e^{2B} + 5B^3e^{B} - 1}\right)$$

The differential Equations (34) and (35) involving $D^2\theta_1$ and $D^2\Phi_1$ gives the solvability condition which is given by

$$\int_0^1 \left(1 + f(z)\right)W_1\,dz = \left(1 + D_F + Sr + Le\right) \tag{40}$$

The expressions for W_1 is back substituted into Eq. (40) and integrated to yield an relation for the critical Rayleigh number and Marangoni number are obtained.

5. RESULTS AND DISCUSSIONS

In this paper, we focused on the combined effects of the viscosity parameter and the internal heat production in the presence of thermo and thermal diffusion parameter on Rayleigh-Benard convection (due to buoyancy) and Marangoni convection (due to surface tension). A linear stability study was performed, the boundaries are considered rigid, but permeable, and insulated to temperature disturbances, and the obtained eigenvalue problem was solved using the

perturbation procedure. In carrying out the analytical computations,

5.1 Rayleigh-Benard Convection

Internal heat generation, Ns and variable viscosity viscosity, B effects is investigated theoretically using perturbation procedure, critical Rayleigh number is obtained and the marginal stability curves in the plane is identified. Marginal stability curves will shift upwards when the value of Ns increases and curves will shift downwards when the value of B increases which state that the internal heating in the fluid layer stabilizes the system and the temperature-dependent viscosity destabilizes the system. From figures 2 and 3, the trends of stability for two parameters which are the thermal diffusion parameter, Sr and thermo diffusion parameter, D_F that exist in a double diffusive is investigated. The stability curves for the effects are shown in figure 2 and figure 3. As can be seen clearly in figure 2, R_c increases when Sr increases and thus enhance the onset of convection due to the decrease of temperature flux. Meanwhile, for Dufour parameter, it shows that an increase of Dufour parameter, D_F will increase the R_c as illustrated in figure 3.

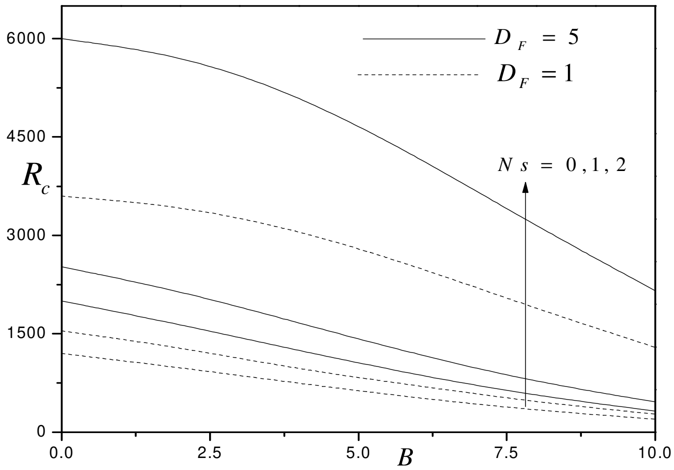

Figure 2: Effects of B, Ns & D_F to Critical Rayleigh Number with $Sr = 0.5$.

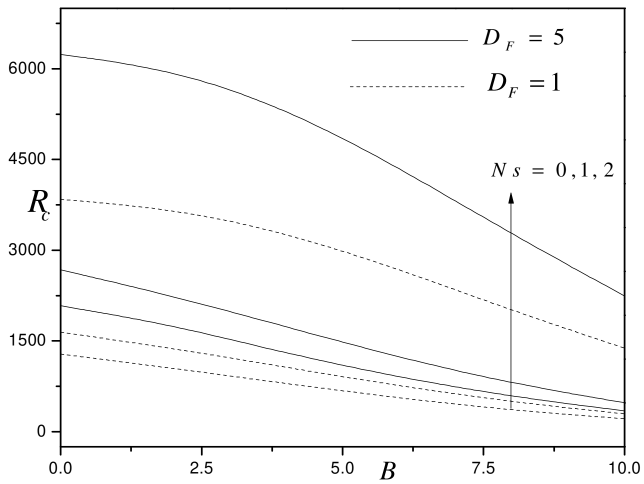

Figure 3: Effects of B, Ns & D_F to Critical Rayleigh Number with $Sr = 1$.

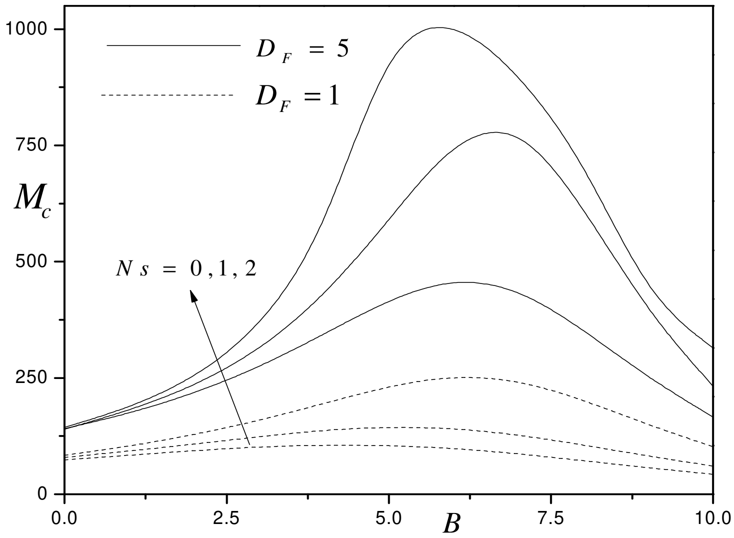

Figure 4: Effects of B, Ns & D_F to Critical Marangoni Number with $Sr = 0.5$.

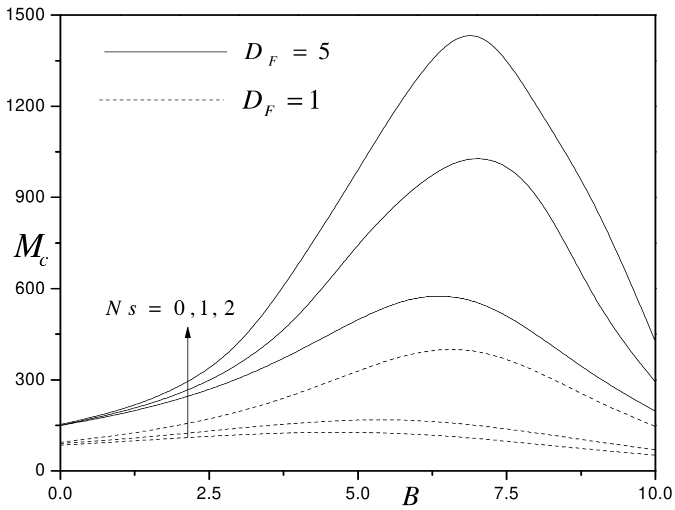

Figure 5: Effects of B, Ns & D_F to Critical Marangoni Number with $Sr = 1$.

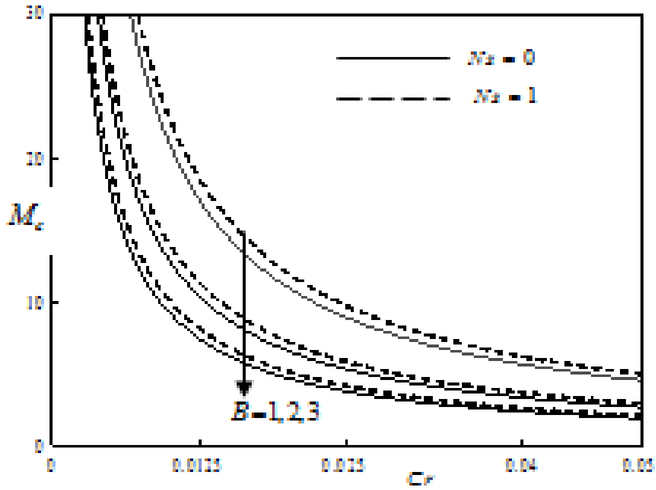

Figure 6: Effects of B, Ns & C_r to Critical Marangoni Number with $Sr = 1$ & $D_F = 1$.

5.2 Marangoni Convection

Marangoni convection (due to surface tension) on the onset of steady convection in a double-diffusive fluid layer is investigated theoretically. Variable viscosity parameter, B and internal heat generation, Ns effects with of Soret and Dufour effects are tested, the critical M_c increases as shown in figure 4 and 5. The trend of stability for the Soret effect, Sr, internal heating source, Ns and Dufour effect, D_F is also shown in the same figures. The variation of increasing the Soret and Dufour effects increases the critical Marangoni number. This observation reveals that in Marangoni convection, variable viscosity parameter, B is to accelerate the arriving of convection and destabilize the system. Meanwhile, Ns, Sr and D_F delays the arriving of convection and stabilize the system. The effect of Crispation number, Cr on the marginal stability curves will shift downwards when the value of B increases and an increase in value of Cr is to decrease the value of M_c and thus making system more unstable as illustrated in figure 6.

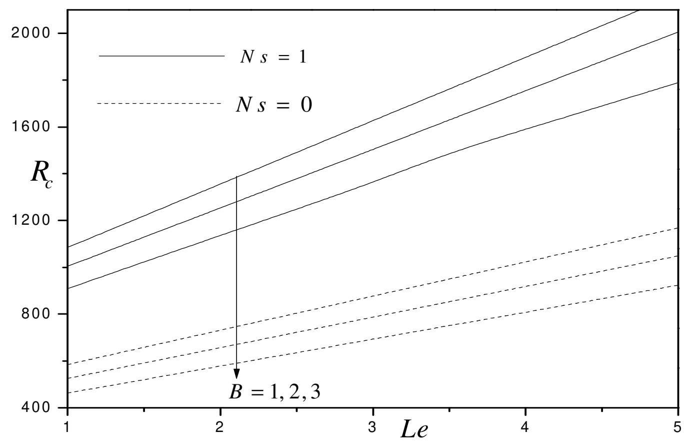

Figure 7: Critical R_c Versus Le for Different Values of

B with $Sr = 1 = D_F$.

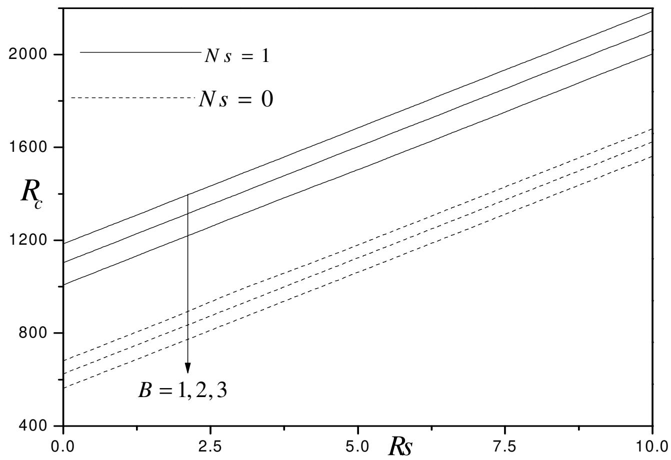

Figure 8: Critical R_c Versus Rs for Different Values of B with $Sr = 1 = D_F$.

5.3 Lewis and Solutal Rayleigh Effect.

Solutal Rayleigh, Rs effect and Lewis number, Le due to the solutal balances in a binary fluid where Le is the ratio between characteristic diffusion of mass and lengths for diffusion of heat and Rs is the Rayleigh number representing the solutal balances. The effects to the system were shown in figure 7 and figure 8. Figure 7 represents the Le effect on the critical Rayleigh number, R_c against Ns and B . It is observed that Le shows the same results as Ns where an increase of the effects values make the system stabilized. The increasing values of Rs , Ns and B is shown together in figure 8. An increase of Rs make the system more stable. From all the figures shown, the system is more stable when Rs , Ns and Le higher values and the system is more unstable for higher values of B.

6. CONCLUSIONS

The combined effect of internal heat generation and variable viscosity parameter in the presence of thermo and thermal diffusion parameters is being examined in this research paper where results show that an increasing value of B accelerates the onset of Rayleigh-Benard convection and opposite effect are shown case of Marangoni for small values of B . The effects Rs , Ns, Le and D_F will decelerates the onset of convection. Rs , Ns, Le and D_F stabilize the system in both type of convection(Rayleigh-Benard and Marangoni).

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